Large Angle-of-Attack Missile Control Concepts for Aerodynamically Controlled Missiles

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A coupled or cross-axis autopilot control concept is presented which can provide a significant increase in the usable range of angle of attack for aerodynamically controlled tactical missiles without an attendent increase in either steering response time or roll channel bandwidth. The unique feature of the coupled control system design is that adequate airframe stability can be maintained in the presence of aerodynamic cross-coupling at large angles of attack without either physically rolling the airframe to a preferred orientation with respect to the airstream or increasing the bandwidth of the roll system. Stability is maintained by intentionally cross-coupling sensor signals among the control channels (i.e., roll sensor signals into steering control, and vice versa) at large angles of attack. In effect, the intentional cross-channel coupling partially cancels the destabilizing effect of the interchannel aerodynamic coupling.

I. Introduction

TO date, control designs for cruciform surface-to-air missiles typically have been based on the use of three independent channels for roll, pitch, and yaw control. However, in actual practice, complete decoupling between the roll and steering (i.e., pitch and yaw) channels generally cannot be achieved due to inherent aerodynamic interactions, which become increasingly significant as the missile angle of attack is increased. In effect, this aerodynamic interaction comprises a dynamic interchannel coupling mode, and the maintenance of stability becomes increasingly more difficult as missile angle of attack is increased.

The tendency toward roll-yaw-pitch (R-Y-P) instability has been circumvented by limiting the maximum angle of attack and by partially decoupling the steering and roll control systems via the use of a roll bandwidth that exceeds the steering bandwidth by a significant factor (e.g., 2 to 4). Unfortunately, effective interception of high-performance air targets can be achieved only via high terminal maneuverability and thus large angle of attack in combination with very fast steering response times. The result has been a continuing effort to maximize roll system bandwidth, together with an acceptance of some reduction in both the steering response and in maximum allowable angle of attack. The limitations on steering performance can lead only to less than optimal performance for the more difficult intercept conditions, whereas efforts to achieve wider and wider roll bandwidths have led to increasingly severe design requirements on control servos, instruments, and aeroelastic characteristics.

Recently, optimal design techniques[1,2] have been adapted successfully to the synthesis of a combined roll/steering system for a surface-to-air missile in which the aerodynamic interactions were considered directly as a part of the total control synthesis problem and the design was constrained by specific requirements on the steering and roll loop response. The objective was to determine the increase in maneuverability (i.e., angle of attack) limits which can be achieved without any increase in the steering response time and roll bandwidth.

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II. Description of Aerodynamic Coupling Problem

Limitations of the maneuverability or angle of attack of rocket-powered, aerodynamially controlled missiles may be determined by any of the following factors: 1) induced moments (pitch, yaw, or roll) that exceed the missile control capabilities; 2) inadequate control effectiveness or excessive control cross-coupling, or 3) structural limits of the airframe. Item 3 establishes the upper bound on the maneuverability of the vehicle. However, this section specifically addresses items 1 and 2, which often limit missile maneuverability, particularly at low dynamic pressure flight conditions, where a high angle of attack is required to achieve the desired maneuvers. Specific design limitations that may result from this R-Y-P coupling will be discussed in terms of their effect on the design of a high-performance, small-winged, tail-controlled, tactical missile; however, the general technical approach is applicable to all cruciform aerodynamic control configurations.

Several flow phenomena are responsible for the aerodynamic characteristics that lead to the R-Y-P coupling problem.[3,4] Vortices are shed from the body and stream back

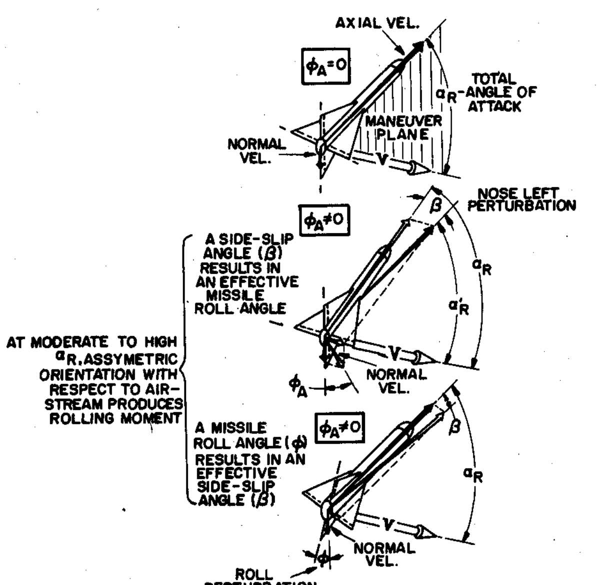

Fig. 1 Aerodynamic coupling due to missile roll in presence of large angles of attack.

over the wings and tail surfaces. Other vortices are shed from wing surfaces and stream back over the tail surfaces. These vortices apply large unwanted torques to the missile. The effects of these vortices increase with angle of attack and depend on the missile roll orientation with respect to the airstream. A substantial reduction in the detrimental effects of the vortices can be accomplished[4] by roll-positioning the missile in the airstream using bank-to-turn control and intedigitated wing and tail surfaces (wings in + and tails in × orientations with respect to the airstream). However, optimum placement of wings and tails to alleviate the R-Y-P coupling problem may not be compatible with overall system design constraints (i.e., launcher and magazine constraints).

Another type of airflow phenomenon responsible for R-Y-P coupling is the strong shock on the windward side which increases the static pressure on the windward side. Under this condition, the lower wing and tail surfaces are immersed in a pressure field that is much stronger than the field that surrounds the upper surfaces. This pressure differential increases with angle of attack.

These aerodynamic effects result in two separate and distinct coupling phenomena that directly impact the selection of autopilot parameters and performance constraints. These are 1) induced roll moments due to change in airframe attitude, and 2) control surface coupling due to the development of unequal control forces for the same deflection of opposing control surfaces.

In the first case, when the missile is at a large "pitch" angle of attack (upper view of Fig. 1), any perturbation in "yaw" will result in much larger side forces on the wing and tail surfaces that are located on the windward or lower part of the body than will result on the surfaces located on the upper part of the body. A nose-left perturbation in yaw therefore will result in a "roll" ϕ_A relative to the airstream, as shown in the middle view of Fig. 1. In addition, an actual roll will result in an effective sideslip angle β and thus in a larger wind force applied to the side of the missile (lower view of Fig. 1). This, in turn, will cause a larger roll action due to the pressure differential between the upper and lower surfaces. Hence, an inherently unstable loop exists "roll-yaw-pitch" (R-Y-P). Even if some aerodynamic configuration modifications are possible to lessen the severity of the R-Y-P coupling, the inequality of windward and leeward energy levels always must be reckoned with.

The induced roll moment due to sideslip angle is a very strong function (i.e., both direction and magnitude) of the orientation of the vehicle with respect to the airstream. It is this particular induced moment that motivates bank-to-turn rather than skid-to-turn control. In particular, when the tails and wings are in line and orientated in a + configuration with respect to the airstream (i.e., $\phi_A = 0$), the induced roll moment is, in general, destabilizing, and as the pitch plane angle of attack increases the destabilizing effect becomes more severe. However, when the tails and wings are oriented in an × configuration with respect to the airstream (i.e., $\phi_A = -45°$), the induced roll moment is, in general, stabilizing. Thus larger angles of attack and consequently greater maneuverability can be achieved by rotating the airframe away from the $\phi_A = 0$ maneuver configuration with respect to the airstream.

With regard to the coupling that is induced by control surface deflection, a typical tail-controlled missile uses two orthogonal tail surface pairs (*a, c* and *b, d:* left side of Fig. 2) for both steering and roll control. Selected pairs of tail surfaces, (*a, c,* and *b, d*) are used for steering control, and all four surfaces are shared for roll control. Typically, the autopilot design is based on the assumption that roll and steering tail deflection produce uncoupled missile motion (i.e., steering tail deflection produces only steering motion, and roll deflection produces only roll motion). This assumption is violated as the angle of attack increases and can result in stability problems due to control cross-coupling. The left-hand part of Fig. 2 shows a missile maneuvering with a

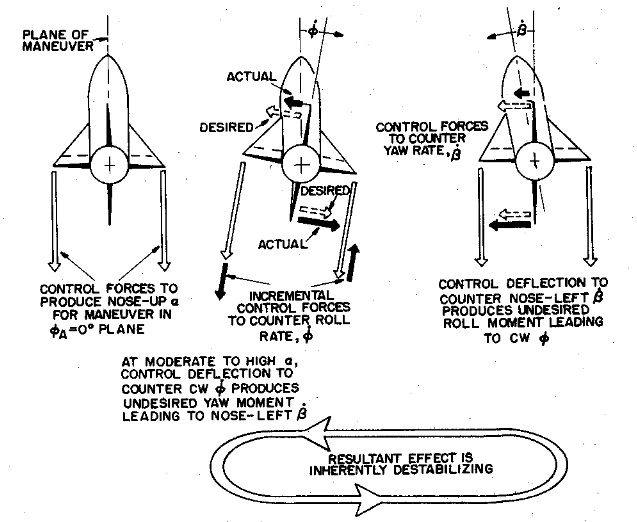

Fig. 2 Aerodynamic coupling due to unequal control surface force coefficients.

high angle of attack. A clockwise perturbation in roll is countered by the roll control system, as shown in the middle of Fig. 2. All four control surfaces are deflected in such a way as to produce an opposing roll torque. The autopilot was designed assuming that no control coupling exists; i.e., if each surface is deflected an equal amount, an equal force will be produced at all four surfaces. The problem is that at high angles of attack the windward tail may develop a much larger force, as shown in Fig. 2. An undesired resultant side force to the right is produced which moves the nose of the missile left. The nose-left action is countered by developing a left side force with the yaw control surfaces. The unequal tail forces, however, roll the missile further in the clockwise direction, leading to an unstable situation, as shown in the right side of Fig. 2.

III. Modeling the Control and Aerodynamic Characteristics

A single flow diagram illustrating existing missile control systems is given in Fig. 3. Here the two steering planes are designated as *A* and *B* planes. Each is implemented independently (i.e., there is no intentional cross-coupling of

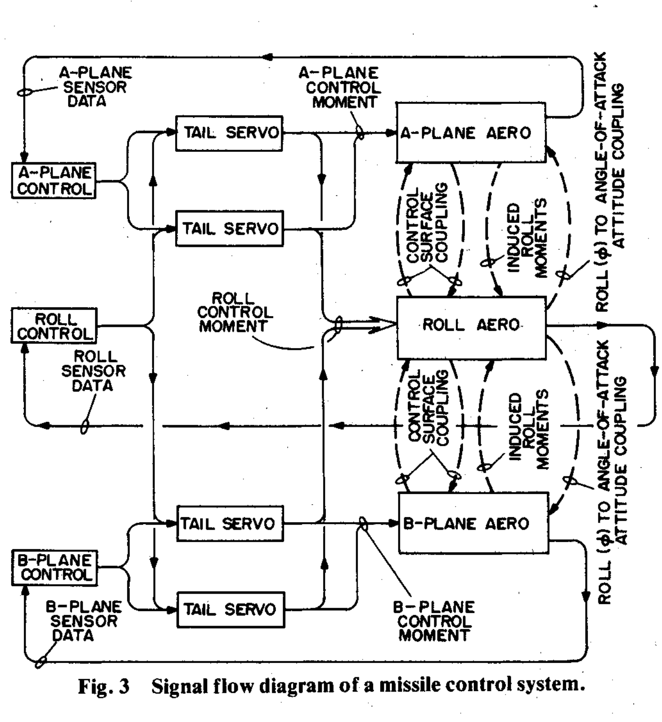

Fig. 3 Signal flow diagram of a missile control system.

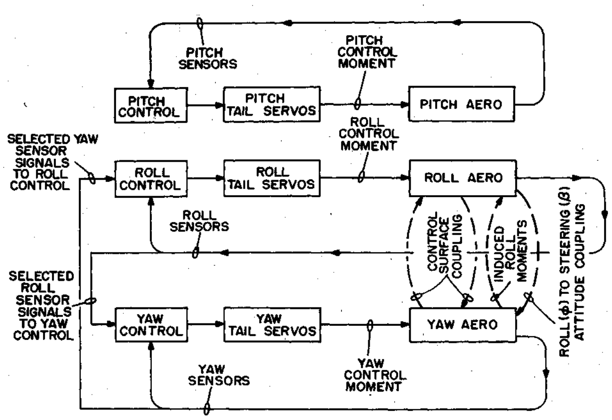

Fig. 4 Functional block diagram of a coupled missile control system.

sensor signals for control). An unintentional cross-coupling exists, as discussed previously, due to control surface deflections and induced roll moments, which are illustrated by the heavy dashed lines in Fig. 3. This particular representation is difficult to analyze because the degree of cross-coupling from each steering plane to roll and vice versa is a very strong function of the plane of maneuver, i.e., the orientation of the vehicle with respect to the airstream.

In order to simplify the analysis of R-Y-P stability, a simplification has been performed which allows the coupling problem to be analyzed as small perturbations about a desired plane of maneuver. The following definitions will be applied:

1) Pitch system: The pitch system is defined here as the equations of motion which define the behavior of the vehicle in the principal plane of maneuver. The plane of maneuver is defined as the plane, which, for zero perturbations, contains the missile centerline and velocity vector.

2) Yaw system: The yaw system is defined here as the equations of motion which define the behavior of the vehicle in a plane perpendicular to the principal plane of maneuver. These are considered to be perturbations in missile attitude.

3) Roll system: The roll system is defined as the equations of motion which define the behavior of the vehicle about the missile centerline. The use of linear analytical procedures is facilitated by considering only small changes in roll orientation. With this definition, small changes in roll attitude produce only negligible changes in effective pitch attitude and may be ignored. However, since the normal yaw attitude is assumed to be near zero, these same changes represent a major change in yaw orientation.

Utilizing the preceding definitions, the missile control system can be simplified for analysis of R-Y-P coupling, as shown in Fig. 4. With this representation, the significant cross-coupling occurs only between the roll and yaw systems. The pitch system, for analysis purposes, can be considered independent of the yaw and roll systems. Also included in Fig. 4 are the signal flow paths for the proposed cross-coupling of sensor signals between the yaw and roll systems. These additional feedback control paths will aid in negating the effects of the inherent aerodynamic cross-coupling indicated by the heavy dashed lines.

The system described by Fig. 4 will be used for design and analysis of the control system. Only when we consider the implementation of the control law will it be necessary to utilize the missile control system representation given in Fig. 3. (i.e., both the A and B planes).

IV. Design Approach

The objective of this section is to outline the design technique used for developing the control policy for stabilizing the effects of R-Y-P coupling. The technique utilized commonly is referred to as the "model-in-the-performance index." The concept was developed by Kalman[1] and later adapted to aircraft control systems by Tyler.[2]

In general, model matching techniques are applicable when desired characteristics of the system can be described by either transfer functions or a set of differential equations. In particular, with the model-in-the-performance index formulations, desired system characteristics such as response times and bandwidths can be specified in the model.

Usually the R-Y-P coupling problem can be circumvented by ensuring that the roll and steering control systems are decoupled essentially by choosing a roll system bandwidth that is very much larger than that of the steering system. Typically this results in either a requirement for an extremely wide bandwidth roll system or an increase in the response time/decreasing bandwidth of the steering system.

An increase in steering response time is not desirable because of its significant impact on guidance performance, whereas the bandwidth of the roll system is limited due to the elastic properties of the airframe, control servo bandwidth, and the effects of noise in the system. Thus, in the design of a missile control system to maintain stability in the presence of R-Y-P coupling, the response time of the steering system must be maintained, whereas the bandwidth of the roll system also must be limited. An undesirable limit on allowable angle of attack often therefore must be imposed to accommodate these conflicting constraints. On the other hand, use of the model-in-the-performance index concepts provides a mechanism for the imposition of these constraints directly into the derivation of the control law in a way that relaxes the requirements on roll loop bandwidth for a given steering response speed and allowable angle-of-attack without loss of control due to R-Y-P coupling.

Error Equation

For the model-in-the-performance index design procedure, let the model and system be defined as

$$\dot{x}_m = Dx_m \tag{1}$$

$$\dot{x} = Ax + Bu \tag{2}$$

where

x_m $= n \times 1$ state vector of the model‡
x $= n \times 1$ state vector of the system
u $= m \times 1$ control vector
D $= n \times n$ model matrix
A $= n \times n$ system matrix
B $= n \times m$ control distribution matrix

Define an error equation as

$$\dot{x} - \dot{x}_m \simeq e = (A - D)x + Bu \tag{3}$$

where e can be interpreted for small errors as the difference between the derivatives of the system and model states as indicated. More importantly, however, is that only the model matrix and not the model states appears in the error equation as defined by the right-hand side of Eq. (3).

To illustrate further the implications of Eq. (3), the error equation may be expressed in terms of the closed-loop system and model equation by imposing a desired control law (4), which is some linear combination of the system states

$$u = Gx \tag{4}$$

where G is a state feedback gain matrix of the appropriate dimension. Substituting the control equation (4) into the error equation (3) yields

$$e = [(A + BG) - D]x \tag{5}$$

‡It is not necessary that the model be the same dimensionally as the system.[1,2]

From Eq. (5), it is clear that any procedure that effects a reduction in the error e must involve an adjustment of the elements of the closed-loop matrix to approach the elements of the model matrix [i.e., $(A+BG) \rightarrow D$ as e is reduced]. As a result, the characteristics (e.g., bandwidth and response time) of the closed-loop system approach those of the model as the error is made to approach zero. Minimization of the error e provides a direct method of limiting the bandwidth of the roll system and maintaining present steering response time while maintaining R-Y-P stability so long as all important longitudinal, lateral, and roll aerodynamic terms are included properly in the system matrix A. Reduction of e is achieved via proper choice of the control matrix G and normally results in the inclusion of compensating control signals between the steering and roll channels, in addition to the normal roll and steering loop formulations.

Performance Measure

To complete the definition of the optimization problem, a cost function must be established. Since it is desired to establish a tradeoff between a reduction in the error e and prevention of excessive control inputs, the following per-

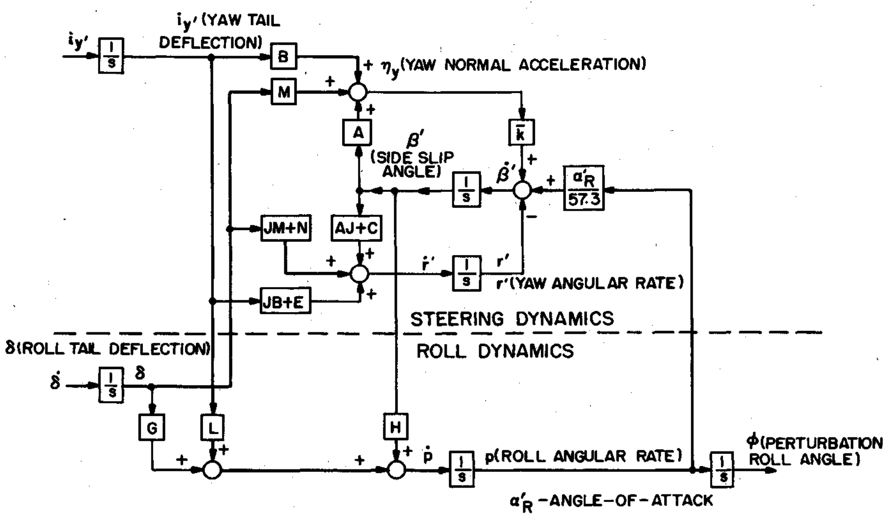

Fig. 5 Block diagram of the linearized yaw and roll system dynamics.

Table 1 Definition of parameters and symbols

Aerodynamic Parameters	Aerodynamic Coefficients and Stability Derivatives	Symbols
$A = \frac{q_\infty S}{W} C_{Y'_{\beta'}}$ (g's/deg)	C_ℓ - roll moment coefficient	c.g. - Center of Gravity (inches)
		d - missile reference length diameter(ft)
$B = \frac{q_\infty S}{W} C_{Y'_{i_{y'}}}$ (g's/deg)	$C_{\ell_{i_y}} = \frac{\partial C_\ell}{\partial i_{y'}}$ (1/deg)	I_x - roll moment of inertia (slug-ft^2)
		$I_{z'}$ - yaw moment of inertia (slug-ft^2)
$C = \frac{(57.3) q_\infty Sd}{I_{z'}} C_{n'_{\beta'}}$ (1/sec^2)		$i_{y'}$ - yaw tail deflection (deg)
		p - roll angular rate (deg/sec)
$E = \frac{57.3 q_\infty Sd}{I_{z'}} C_{n'_{i_{y'}}}$ (1/sec^2)	$C_{\ell_{\beta'}} = \frac{\partial C_\ell}{\partial \beta'}$ (1/deg)	q_∞ - dynamic pressure for free stream conditions (lb/ft^2)
		r' - yaw angular rate (deg/sec)
$G = \frac{57.3 q_\infty Sd}{I_x} C_{\ell_\delta}$ (1/sec^2)	$C_{n'}$ - yawing moment coefficient	S - missile reference area (cross - section) (ft^2)
		V_m - missile velocity (ft/sec)
$H = \frac{57.3 q_\infty Sd}{I_x} C_{\ell_{\beta'}}$ (1/sec^2)	$C_{n'_{i_{y'}}} = \frac{\partial C_{n'}}{\partial i_{y'}}$ (1/deg)	W - weight (lbs)
		α - angle of attack (deg)
$J = \frac{57.3 W (cg - cg_{REF})}{12 I_{z'}}$ (deg/(g's-sec^2))	$C_{n'_{\beta'}} = \frac{\partial C_n}{\partial \beta'}$ (1/deg)	α'_R - angle-of-attack for zero perturbations of the yaw system (deg)
$\bar{k} = 1845/Vm$ (deg/(g's-sec))	$C_{n'_\delta} = \frac{\partial C_{n'}}{\partial \delta}$ (1/deg)	α_T - trim angle of attack (zero pitch moment) (deg)
		α_R - Total angle of attack (deg)
$L = \frac{57.3 q_\infty Sd}{I_x} C_{\ell_{i_{y'}}}$ (1/sec^2)	$C_{Y'}$ - Yaw force coefficient	β' - side slip angle (deg)
	$C_{Y'_{i_{y'}}} = \frac{\partial C_{Y'}}{\partial i_{y'}}$ (1/deg)	δ - roll tail deflection (deg)
$M = \frac{q_\infty S}{W} C_{Y'_\delta}$ (g's/deg)		η_y - yaw normal acceleration (g's)
	$C_{Y'_{\beta'}} = \frac{\partial C_{Y'}}{\partial \beta'}$ (1/deg)	ϕ - perturbation roll angle (deg)
		α_A - aerodynamic roll angle (deg)
$N = \frac{57.3 q_\infty Sd}{I_{z'}} C_{n'_\delta}$ (1/sec^2)	$C_{Y'_\delta} = \frac{\partial C_{Y'}}{\partial \delta}$ (1/deg)	ϕ_I - Inertial roll angle (deg)

formance index will be used:

$$J=\frac{1}{2}\int_0^t f(e^TQe+u^TRu)\mathrm{d}t \tag{6}$$

Substituting Eqs. (4) and (5) into the performance index (6) yields

$$J=\frac{1}{2}\int_0^t f\,\mathrm{tr}\{[((A+BG)-D)^TQ((A+BG)-D)$$
$$+G^TRG]xx^T\}\mathrm{d}t$$

Where tr[·] is the trace operator. For convenience, let $X=x\,x^T$. The performance index now can be written in the following form:

$$J=\frac{1}{2}\int_0^t f\mathrm{tr}\{[((A+BG)-D)^TQ((A+BG)-D)$$
$$+G^TRG]X\}\mathrm{d}t \tag{7}$$

To complete the problem definition in the framework posed by Eq. (7), the constraint equation may be defined in terms of a state matrix equation as $\dot{X}=\dot{x}x^T+x\dot{x}^T$. Substituting the system differential equation (2) into the preceding equation yields

$$\dot{X}=[A+BG]X+X[A+BG]^T \tag{8}$$

Thus the optimization problem is to determine the elements of G which will minimize the cost function J subject to the constraint equation (8). The matrix minimum principle[5] is applicable to the optimization problem just defined.

V. Design of a Coupled Autopilot

System Dynamics

An analytical description of the system dynamics (i.e., roll and yaw dynamics) shown in Fig. 4 is given in Fig. 5, and an equivalent state vector representation of the system is given by Eq. (9). It should be noted that this representation includes the principal coupling effects for the airframe configuration considered. Nonlinear effects have been ignored, and it should be recognized that the model may require modification for other airframe configurations. The elements of A are independent of time due to the perturbation analysis about the maneuver plane discussed in Sec. III. Table 1 defines the aerodynamic parameters and symbols:

$$\dot{x}=Ax+Bu$$

$$\begin{bmatrix}\dot{r}'\\ \dot{\beta}'\\ \dot{p}\\ \dot{\phi}\\ i_{y'}\\ \dot{\delta}\end{bmatrix}=\begin{bmatrix}0 & (JA+C) & 0 & 0 & (JB+E) & (JM+N)\\ -1 & (\bar{k}A) & \alpha_R'/57.3 & 0 & (B\bar{k}) & (M\bar{k})\\ 0 & H & 0 & 0 & L & G\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}r'\\ \beta'\\ p\\ \phi\\ i_{y'}\\ \delta\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}i_{y'}\\ \dot{\delta}\end{bmatrix} \tag{9}$$

The validity of this linearized representation of R-Y-P coupling has been established through closed-loop system studies. Typically R-Y-P coupling instabilities occur at relatively low frequencies. Thus, low-frequency approximations to the roll and yaw control systems can be employed which significantly simplify analytical studies, with little effect on the coupled stability margins and response characteristics. The conventional autopilot or control law can be represented as a linear combination of the states given as

$$u=\begin{bmatrix}i_{y'}\\ \dot{\delta}\end{bmatrix}=\begin{bmatrix}K_{11} & K_{12} & 0 & 0 & K_{15} & 0\\ 0 & 0 & K_{23} & K_{24} & 0 & K_{26}\end{bmatrix}\begin{bmatrix}r'\\ \beta'\\ p\\ \phi\\ i_{y'}\\ \delta\end{bmatrix} \tag{10}$$

where the low-frequency approximation has been used to eliminate all but the essential terms from this expression. It should be noted in the conventional autopilot control law [i.e., Eq. (10)] that the yaw and roll control signal channels are not coupled. That is, the signals that determine yaw control tail deflection rate $i_{y'}$ are only a function of the yaw states (r', β', $i_{y'}$), and the signals that determine roll control tail deflection rate ($\dot{\delta}$) are likewise only a function of the roll states (p, ϕ, δ). The linearized representation of the closed-loop system defined by (9) and (10) has been used with considerable success for investigating actual flight instabilities, as well as the instabilities predicted from a six-degree-of-freedom simulation. In particular, the linearized analysis has been slightly conservative. That is, the predicted angle of attack for a R-Y-P instability from linearized analysis is typically less than that observed from actual flights or from the six-degree-of-freedom simulation.

Coupled Control Law

One problem does occur in modeling the system, as shown by Eq. (9), and that is that the feedback gains K must alter the elements of the resultant closed-loop system matrix $(A+BK)$ which are a function of the aerodynamic geometric cross-coupling. In addition, the number of rows of A in which critical cross-coupling elements appear should not exceed the number of control inputs (i.e., two for the system considered here). Otherwise a good match between model and system cannot be guaranteed.

Observing Eq. (9), we see that with this formulation the control can change only the last two rows of the $A+BK$ matrix. However, the aerodynamic cross-coupling terms appear in the first three rows, which cannot be changed by the feedback control law. Since the control cannot alter the rows of the $A+BK$ matrix which contain the cross-coupling terms, it therefore is not possible to obtain a match between system and model as defined by the model-in-the-performance index design technique.

Fortunately, a change of variables yields a more desirable form. Defining the new state vector Z,

$$Z^T=[\eta_{y'}\ \dot{r}'\,r'\ \dot{p}\ p\ \phi]$$

where

η_{y}' = yaw normal acceleration, g
$\dot{r}'$ = yaw angular acceleration, deg/sec^2
r' = yaw angular rate, deg/sec
$\dot{p}$ = roll angular acceleration, deg/sec^2
p = roll angular rate, deg/sec
ϕ = roll angle, deg

results in the following differential system equation:

$$\dot{Z}=A_zZ+B_zu$$

$$\begin{bmatrix} \dot{\eta}_{y'} \\ \ddot{r}' \\ \dot{r}' \\ \ddot{p} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A\bar{k} & 0 & -A & 0 & A\alpha_R'/57.3 & 0 \\ \bar{k}(AJ+C) & 0 & -(AJ+C) & 0 & (AJ+C)\alpha_R'/57.3 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \bar{k}H & 0 & -H & 0 & H\alpha_R'/57.3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \eta_{y'} \\ \dot{r}' \\ r' \\ \dot{p} \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} B & M \\ (JB+E) & (JM+N) \\ 0 & 0 \\ L & G \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_y \\ \dot{\delta} \end{bmatrix} \tag{11}$$

The preceding transformation groups all terms involving control surface forces and moments into the control distribution matrix (B_z), whereas the plant matrix (A_z) contains only terms that are body-originated aerodynamic forces and moments. As a result, the two principal aerodynamic cross-coupling effects, namely, induced roll moments due to sideslip angle (H) and control surface deflection cross-coupling (L, N, M), are separated clearly.

In this formulation, the elements of the control matrix directly influence the three rows of the A_z matrix which are a function of the aerodynamic and geometric cross-coupling. However, there are three rows containing cross-coupling elements and only two control inputs. That is, since the system is sixth order and the control distribution matrix has three independent rows, 18 feedback gains are necessary for an exact match of the model and system, but only 12 gains can be implemented because only two independent control inputs are available. Thus, an exact match between model and system still is not possible. Re-examining Eq. (11), an additional characteristic of this set of equations should be noted. Of the three rows containing cross-coupling elements (i.e., 1, 2, and 4), rows 2 and 4 contain the cross-coupling elements for the rotational behavior of the vehicle, and row 1 contains the cross-coupling elements for the translational equations of motion. Since the R-Y-P coupling instability is principally a rotational phenomenon, it is essential that the control law have independent control of rows 2 and 4. In addition, it should be noted that the magnitudes of the first row elements B and M of the control distribution matrix (i.e., normal force resulting from yaw and roll tail deflections, respectively) are small compared to the other elements of the B_z matrix. As a result, a control law designed to match rows 2 and 4 should have little effect on the first row of the closed-loop system matrix [i.e., (A_z+B_zG)]. On the other hand, the only cross-coupling element appearing in the first row of the system matrix at large angles of attack is $A\alpha_R'/57.3$. This is a geometric coupling from roll rate to normal acceleration. For the R-Y-P coupling problem considered here, there is no corresponding coupling from translation motion to the roll system forming a closed-loop coupled interaction (i.e., this coupling element results only in a perturbation of normal acceleration), and typically the effect of this term is small. Thus, even though an exact match of the system and model is not possible, a very good approximation can be achieved.

The model, as discussed earlier, is the low-angle-of-attack closed-loop roll and steering systems. Neglecting the cross-coupling terms in Eq. (11), the model is given by

$$\begin{bmatrix} \dot{\eta}_{y'} \\ \ddot{r}' \\ \dot{r}' \\ \ddot{p} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A\bar{k} & 0 & -A & 0 & 0 & 0 \\ \bar{k}(AJ+C) & 0 & -(AJ+C) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \eta_{y'} \\ \dot{r}' \\ r' \\ \dot{p} \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} B & 0 \\ (JB+E) & 0 \\ 0 & 0 \\ 0 & G \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_{y'} \\ \dot{\delta} \end{bmatrix} \tag{12}$$

where the low-frequency approximation for the control law (yaw and roll tail rate deflections) is defined as

$$\begin{bmatrix} i_{y'} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & g_{24} & g_{25} & g_{26} \end{bmatrix} \begin{bmatrix} \eta_{y'} \\ \dot{r}' \\ r' \\ \dot{p} \\ p \\ \phi \end{bmatrix}$$

Thus, for the model equation $\dot{Z}_m = D\,Z_m$,

$$D=\begin{bmatrix} A\bar{k}+Bg_{11} & Bg_{12} & -A+Bg_{13} & 0 & 0 & 0 \\ (JB+E)g_{11}+(AJ+C)\bar{k} & (JB+E)g_{12} & (JB+E)g_{13}-(AJ+C) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Gg_{24} & Gg_{25} & Gg_{26} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{13}$$

The coupled control law therefore is obtained by substituting Eqs. (11) and (13) into Eqs. (7) and (8) and minimizing (7) subject to (8). The resulting control law is defined by Eq. (14) and, as indicated previously, cross-couples steering signals into roll, and vice versa. The cross-coupling exists whenever the underlined k's are nonzero:

$$\begin{bmatrix} i_{y'} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & \underline{k_{14}} & \underline{k_{15}} & \underline{k_{16}} \\ \underline{k_{21}} & \underline{k_{22}} & \underline{k_{23}} & k_{24} & k_{25} & k_{26} \end{bmatrix} \begin{bmatrix} \eta_{y'} \\ \dot{r}' \\ r' \\ \dot{p} \\ p \\ \phi \end{bmatrix} \tag{14}$$

VI. Application

Comparison of R-Y-P Stability Margins

For all cases considered in this investigation, the main emphasis was on minimizing the error between the system and model, and, as a consequence, the weighting on the control signal in the performance index [see Eq. (6)] had a negligible influence on the resulting control law. That is, the best match between model and system achievable with the model-in-the-performance index approach was utilized. This type of design procedure yields time responses and eigenvalues of the closed-loop system which are nearly exact duplicates of the model. Thus, the primary differences between the model and resulting systems are the performance degradations due to parameters variations.

One method of assessing the impact of parameter variations on system performance is to evaluate the stability margins associated with critical parameters. Fig. 6 compares the minimum stability margins for a conventional design and a coupled control system as a function of angle of attack. For the coupled system, the minimum stability margin decreases from 11 to 3 dB at the largest angle of attack considered. On the other hand, the margin for the existing system, although stable at low angles of attack, becomes very unstable as the angle of attack increased. These results indicate that application of the coupled control concept can increase significantly missile maneuverability previously restricted by R-Y-P instabilities.

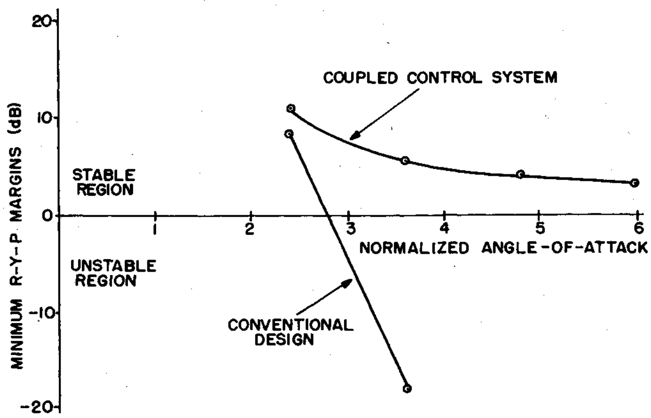

Fig. 6 Comparison of minimum roll-yaw-pitch stability margins vs angle of attack.

Identification of Critical Control Law Parameters

For any practical implementation task, it is of interest to determine if any of the control cross-coupling feedback gains can be eliminated without significantly degrading performance. Moreover, implementation problems also are eased if the steering and roll loop self-feedback gains (k_{11}, k_{12}, k_{13}, k_{24}, k_{25}, and k_{26}) can remain constant with changes in angle of attack. To this end, the following procedure was employed for the modification of the optimal feedback gain schedule:

1) If the cross-coupling feedback gains had a negligible effect on the minimum R-Y-P stability margin and the perturbation response characteristics were not altered significantly by their absence, then these cross-coupling

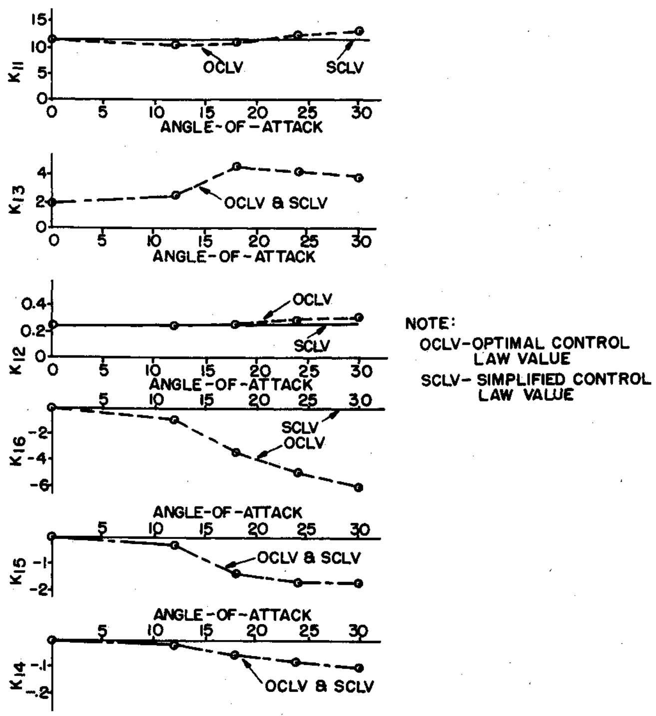

Fig. 7 Yaw tail rate control signal gains vs angle of attack.

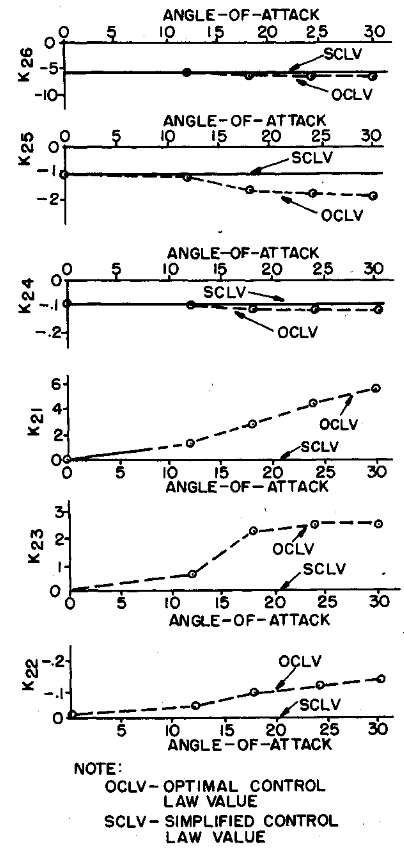

Fig. 8 Roll tail rate control signal gains vs angle of attack.

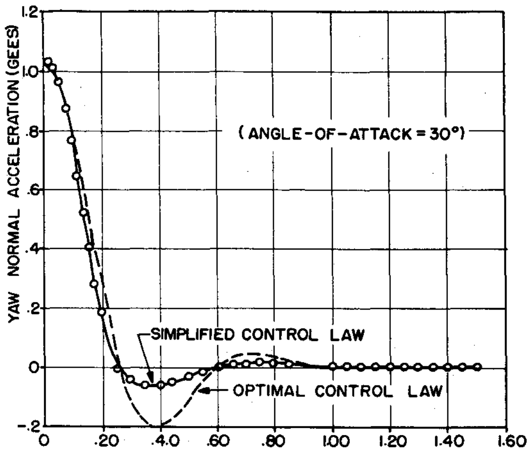

Fig. 9 Yaw response due to an initial 1-*g* acceleration.

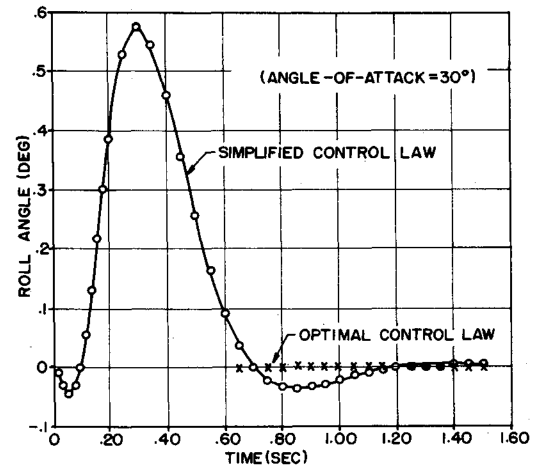

Fig. 10 Roll angle response due to an initial 1-*g* acceleration.

feedback gains are not critical for maintaining system performance and can be set to zero.

2) If approximating the self-feedback gain schedule by a constant (i.e., low angle-of-attack value) had a negligible effect of the minimum R-Y-P stability and system response characteristics, then this simplified feedback gain schedule is recommended for implementation.

The resulting gains values are compared with the optimal values as a function of angle of attack in Figs. 7 and 8 for the yaw and roll control law, respectively, and the performance of the optimal and simplified control configurations is compared in Figs. 9 and 10. The difference between the normal acceleration response characteristics is considered negligible. The amount of induced roll motion of the simplified control law due to a perturbation of the steering system should be acceptable. There was a negligible difference between the stability margins of the simplified and optimal control laws. Therefore, Fig. 6 applies to both control laws.

In summary, only two cross-coupling gains (k_{14}, roll angular acceleration to yaw tail rate; and k_{15}, roll angular rate to yaw tail rate) have a significant effect on the R-Y-P stability margins. Also, all self-feedback gains can be held constant at their low angle-of-attack values except k_{13} (yaw angular rate to yaw tail rate). This gain increases by approximately a factor of 2 at large angles of attack to compensate for changes in the longitudinal aerodynamic stability characteristics.

Further simplification of the control law may be possible, particularly for the feedback gains as a function of angle of attack. This additional simplification was not warranted here because of the limited number of flight conditions examined. However, such simplifications are deemed advisable for an actual flight control system design.

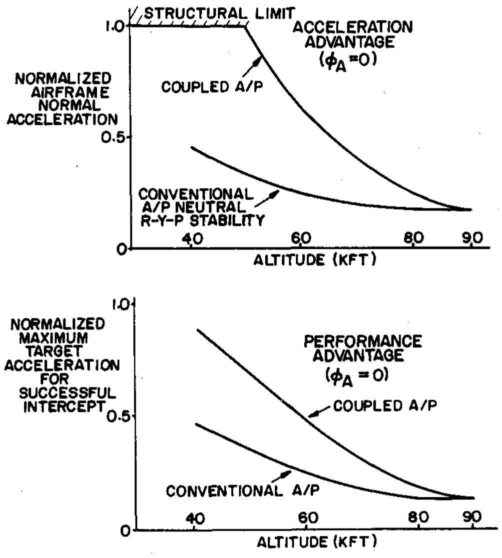

Fig. 11 Comparison of coupled and conventional autopilots.

Comparison of Conventional and Coupled Autopilots

Figure 11 shows the normal acceleration advantage of the coupled autopilot over the conventional uncoupled autopilot. Removal of the R-Y-P coupling stability constraint allows acceleration magnitudes that are limited by structural limitations at low altitudes and control surface angular limitation at high altitudes. The impact of the acceleration advantage is shown by the corresponding performance advantage in Fig. 11. The missile now can intercept targets having greater maneuverability over a wide range of altitudes.

VII. Conclusions

A general and systematic procedure has been devised for the design of highly coupled systems to meet specific response time objectives. The procedure is applicable directly to tactical missile control systems, and the results indicate that, with appropriate signal cross-coupling between control channels, the missile maneuverability can be increased significantly compared to conventional design without experiencing an R-Y-P instability. Maneuvering response times can be maintained without an attendant increase in roll channel bandwidth.

The cross-coupling also may provide additional benefits, such as reduced roll system perturbations due to steering maneuvers. Another potential application of this type of control configuration is to provide a relaxation of roll bandwidth requirements for a given steering response time and maneuverability requirement.

Acknowledgment

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